

Iteration, Substitution and Irreducibility of Polynomials

Mohamed Ayad

May 2, 2008

The purpose of my talk will be to survey two problems in polynomials and to mention related open questions.

First problem Let K be a field and let $f(x) \in K[x]$ be an irreducible polynomial. Set $f_1(x) = f(x)$ and $f_n(x) = f \circ f \circ \dots \circ f(x)$, n times. This polynomial is said to be *stable* if $f_n(x)$ is irreducible over K for any integer $n \geq 1$. Some examples of this kind of polynomials are known, but open questions about this problem remain open.

Second problem Let $F(x, y) \in K[x, y]$ be an irreducible polynomial. It is said to be an *hereditarily irreducible polynomial* (H. I. P) if $F(u(x), v(y))$ is irreducible over K for any pair of nonconstant polynomials $(u(x), v(y))$. All known examples of this kind of bivariate polynomials have their leading terms relatively to (at least) one variable depending on the other variable. So it would be of interest to construct an H. I. P of the form $F(x, y) = ax^n + by^m + \sum a_{ij}x^i y^j$, where a, b and $a_{ij} \in K$ and the sum runs over some couples (i, j) such that $i \leq n$ and $j \leq m$.