A nilpotent quotient algorithm for certain infinitely presented groups

René Hartung

Institute of Computational Mathematics

University of Braunschweig

(joint work with L. Bartholdi and B. Eick)

Let G be a finitely generated group, generated by $\mathcal{X}$ say. The group G is called *finitely L-presented*, if it satisfies

$$G \cong \left\langle \mathcal{X} \ \middle|\ \mathcal{Q} \cup \bigcup_{\varphi \in \Phi^*} \mathcal{R}^\varphi \right\rangle$$

where both $\mathcal{Q}$ and $\mathcal{R}$ are finite subsets of the free group F on $\mathcal{X}$ and Φ^* denotes the free monoid which is finitely generated by some $\Phi \subseteq \mathrm{End}(F)$.

Well-known examples of finitely L-presented groups, which are not finitely presented, are the Grigorchuk group and the Gupta-Sidki group. Both groups are famous for their role in the study of the Burnside problems.

We developed an algorithm for computing polycyclic presentations for the lower central series quotient $G/\gamma_i(G)$, with $i \geq 1$, of a finitely L-presented group G, see [1]. Our algorithm generalizes the nilpotent quotient algorithm for finitely presented groups by Nickel.

An implementation of our algorithm in the computer algebra system GAP, see [2], allows detailed studies of several self-similar groups including the Grigorchuk (super-) group, the Gupta-Sidki group and generalizations of it, the Brunner-Sidki-Vieira group, several generalizations of the Fabrykowski-Gupta group, and the Basilica group. We discovered interesting periodicities in the lower central series of these groups.

References

[1] L. Bartholdi, B. Eick, and R. Hartung. A nilpotent quotient algorithm for certain infinitely presented groups and its applications. Submitted to *Internat. J. Algebra Comput.*

[2] R. Hartung. NQL – *Nilpotent quotients of L-presented groups*, 2007. A submitted GAP package.