ON CURVATURE CONSTRUCTIONS OF SYMPLECTIC FORMS OVER SYMMETRIC SPACES(ABSTRACT)

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In symplectic geometry, it is important to have a method of constructing symplectic forms on total spaces of bundles in a way, that the constructed form restricts symplectically on the fibers (note that it is not required that the base of the bundle is symplectic as well). Such forms constitute a particular class of *coupling forms* which have numerous applications in mathematical physics. One of such methods was proposed by Sternberg and Guillemin. Let there be given a fiber bundle

$$F \to E \to B$$

such that its structure group is a Lie group G. Assume that F admits a G-invariant symplectic structure. If the G-action on F is hamiltonian, with moment map μ, then, the following assumption allows one to construct a symplectic form on E: there exists a connection in the associated principal bundle

$$G \to P \to B$$

such that the connection form Ω restricted to the horizontal distribution of the connection, has the property that the 2-form

$$\langle \Omega(X,Y), \mu(f) \rangle$$

is non-degenerate for all horizontal vector fields X, Y, and all covectors from $\mu(F) \subset \mathfrak{g}^*$.

However, such connections are scarce, and there are obstructions to their existence. Nevertheless, Lerman constructed such connections in fiber bundles which are bundles of coadjoint orbits of compact Lie groups over coadjoint orbits. In the present paper we give a full solution to the following problem: find all fiber bundles of the form

$$H/V \to K/V \to K/H$$

for compact Lie groups K such that the canonical invariant connection has the required property. In particular, new homogeneous bases with the required property have been found, among them locally symmetric spaces of non-compact type, and their locally symmetric compact quotients, homogeneous quaternionic-Kähler manifolds, and some other important classes.

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