Title: Double centralizers of unipotent elements in semisimple algebraic groups

We present the main results appearing in a recent article with R. Lawther, in which we determine the dimension of $C_G(C_G(u))$, for G a simple algebraic group defined over an algebraically closed field of characteristic 0 or of good positive characteristic p, and $u \in G$ a unipotent element.

In characteristic 0, or in case u has order p in positive good characteristic, Seitz has shown that there exists a 1-dimensional connected unipotent subgroup $U \leq G$ with $u \in U$ and such that $C_G(u) = C_G(U) = C_G(\mathrm{Lie}(U))$, and discusses the unicity of such a subgroup.

When u does not have order p (and hence does not lie in a 1-dimensional connected subgroup), one can still hope to find a canonically defined connected abelian unipotent subgroup X of G, with $u \in X$, satisfying the same properties, i.e. $C_G(u) = C_G(X) = C_G(\mathrm{Lie}(X))$. A candidate for X would be the connected component of the double centralizer of u in G, that is, $(C_G(C_G(u)))^\circ = (Z(C_G(u))^\circ$, from which stems our interest in this subgroup.

We will indicate the techniques used in the proofs of our results, and conclude with some remarks about the case of bad positive characteristic, which is under current investigation by my PhD student I. Simion.