Multiplicity-free actions induced by Dual Pairs

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Symmetric Spaces and their Generalizations - June 26, 2012

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(G, V) is *super multiplicity-free* if for any $\Gamma \in Irr(G)$,

$$\dim \mathrm{Hom}_G(P(V), \Gamma) \leq 1.$$

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(G, V) (S)MF $\Leftrightarrow \forall k$: $\dim \mathrm{Hom}_G(P^k(V), \Gamma) \leq 1$

Series of irreducible (S)MF representations

	MF	*SMF*
(1)	$\mathrm{GL}_n \otimes \mathrm{GL}_m$ $(n \geq 0, m \geq 1)$	$\mathrm{GL}_n \otimes \mathrm{GL}_m$ $(n \geq 0, m \geq 1)$
(2)	$\mathrm{GL}_k \otimes \mathrm{Sp}_n$ $(k = 0, 1, 2, 3)$	$\mathrm{GL}_k \otimes \mathrm{O}_n$ $(k = 0, 1, 2, 3)$
(3)	$\mathrm{GL}_1 \otimes \mathrm{O}_n$ $(n \geq 2)$	$\mathrm{GL}_1 \otimes \mathrm{Sp}_n$ $(n \geq 2)$
(4)	$\mathrm{GL}_n \otimes \mathrm{Sp}_4$ $(n \geq 2)$	$\mathrm{GL}_n \otimes \mathrm{Sp}_4$ $(n \geq 2)$
(5)	$S^2\mathrm{GL}_n$ $(n \geq 2)$	$S^2\mathrm{GL}_n$ $(n \geq 2)$
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- Further parallelism in the structure?

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$\Gamma^{(a,b)}$ operators of order $a - b$.

$\Gamma = \Gamma^{(2,0)} \oplus \Gamma^{(1,1)} \oplus \Gamma^{(0,2)}$ Lie s.a. w.r.t. super commutator.

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iv) $(G, \Gamma^{(1,1)})$ is a dual pair on $\mathcal{H}$

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Let I_0^ν denotes the G-invariants of degree ν. By iii), $P^k(V)$ is a quotient of

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$\forall k$: $\tilde{P}^k(V)$ mf. as $G \times \Gamma^{(1,1)}$ mod. $\Rightarrow (G \times G^{(1,1)}, V)$ (S)MF.

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Consider special cases $p = 0$ or $q = 0$:

'Duality' of dual pairs

(G, Γ) on $P(V_0) = S(V_0) \rightsquigarrow (G^\vee, \Gamma^\vee)$ on $P(V_1) = \bigwedge(V_1)$, s. th.

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In this case, we call (G, Γ) and $(G^\vee, \Gamma^\vee)$ a *couple of dual pairs*.

Couples for $V = \mathbb{C}^n \otimes \mathbb{C}^p$

	$S(\mathbb{C}^n \otimes \mathbb{C}^p)$	$\bigwedge(\mathbb{C}^n \otimes \mathbb{C}^q)$
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(3)	$(\mathrm{O}_n, \mathfrak{sp}(2p))$	$(\mathrm{Sp}_n, \mathfrak{sp}(2p))$	$\mathrm{O}_n \otimes \mathbb{C}^\times$	$\mathrm{Sp}_n \otimes \mathbb{C}^\times$

Corresponding (S)MF spaces

	(G, Γ)	$(G^\vee, \Gamma)$	MF	SMF
(1)	$(\mathrm{GL}_n, \mathfrak{gl}(p))$	$(\mathrm{GL}_n, \mathfrak{gl}(p))$	$\mathrm{GL}_n \otimes \mathrm{GL}_p$	$\mathrm{GL}_n \otimes \mathrm{GL}_p$
(2)	$(\mathrm{Sp}_n, \mathfrak{so}(2p))$	$(\mathrm{O}_n, \mathfrak{so}(2p))$	$\mathrm{Sp}_n \otimes \mathrm{GL}_p$	$\mathrm{O}_n \otimes \mathrm{GL}_p$
(3)	$(\mathrm{O}_n, \mathfrak{sp}(2p))$	$(\mathrm{Sp}_n, \mathfrak{sp}(2p))$	$\mathrm{O}_n \otimes \mathbb{C}^\times$	$\mathrm{Sp}_n \otimes \mathbb{C}^\times$

(1) $p \in \mathbb{N}$; (2) $p = 0, 1, 2, 3$; (3) $p = 1$.

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$$S(V) = \bigoplus_{\lambda \in \Xi} \varepsilon_\lambda \cdot V_{\lambda_1} \otimes V_{\lambda_2} \text{ as a } G \times G^{(1,1)} \text{ module.}$$

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Recall: $\tilde{P}^k(V) = \bigoplus_j I^{2j} \otimes \mathcal{H}^{k-2j}$.

Decomposition of harmonics (Cheng, Zhang)

Let $\mathbf{\frac{1}{2}} = (\frac{1}{2}, \ldots, \frac{1}{2}; -\frac{1}{2}, \ldots, -\frac{1}{2})$, $Par(p|q) = \{\lambda : \lambda_{p+1} \leq q\}$.

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$(\mathrm{O}_n, \mathfrak{spo}(2p|2q))$ harmonics:

$$\mathcal{H} = \bigoplus_{\lambda \in Par(p|q),\ \lambda_1^t + \lambda_2^t \leq n} V_\lambda^n \otimes V_{\lambda + n\mathbf{\frac{1}{2}}}^{p|q}$$

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Labelling of $\mathfrak{gl}(p|q)$ representations

$\mathfrak{h} = \sum_{i=1}^{p+q} \mathbb{C} E_{i,i}$ Cartan, $\mathfrak{b} = \sum_{i \leq j} \mathbb{C} E_{i,j}$ Borel subalgebra

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Decomposition of harmonics - Case (2)

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$$\mathcal{H}^k = \bigoplus_{\lambda \in Par(p|q),\ \lambda_1^t + \lambda_2^t \leq n,\ |\lambda| = k} V_\lambda^n \otimes V_\lambda^{p|q}$$

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Decomposition of Invariants - Case (2)

$$I^{2j}_{\mathrm{O}_n} = \bigoplus_{|\mu|=2j,\ \ell(\mu)\leq q,\ \mu_i\leq n,\ \mu'_i\equiv 0(2)} V^q_\mu$$

$$I^{2j}_{\mathrm{Sp}_n} = \bigoplus_{|\mu|=2j,\ \ell(\mu)\leq min(q,n),\ \mu'_i\equiv 0(2)} V^q_\mu$$

as $\mathfrak{gl}(q)$ modules. In particular, $I^{2j}_{\mathrm{O}_n} = I^{2j}_{\mathrm{Sp}_n}$ for $n >> j$.

Decomposition under $H \times \mathfrak{gl}(p)$

In cases (1) - (3), the $H \times \mathfrak{gl}(p)$ decomposition of

$$\tilde{P}^k(V) = \bigoplus_j I^{2j} \otimes \mathcal{H}^{k-2j}$$

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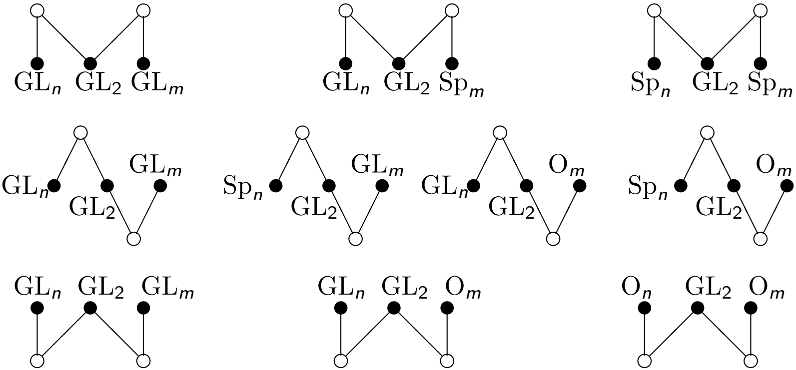
$(\mathrm{SL}_n \otimes \mathrm{Sp}_4)$ (S)MF!

Super MF spaces ($G = G_1 \times G_2 \times G_3$)



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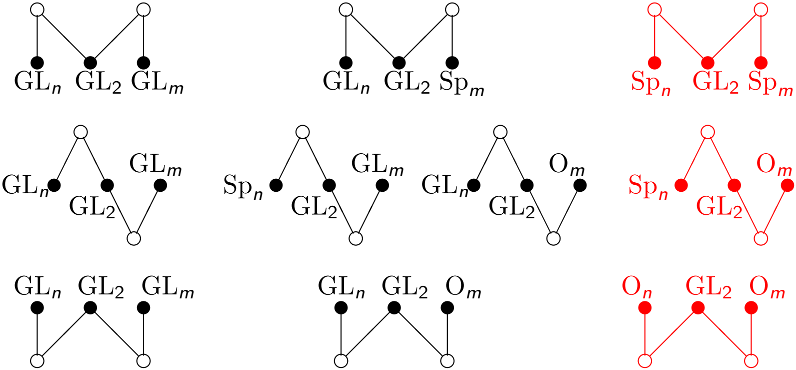

Corresponding couple

Case (2):

$$P^k(V) = \bigoplus_{(\lambda_1,\lambda_2)\in\Xi} V_{\lambda_1} \otimes V_{\lambda_2} \quad (G \times \mathrm{GL}_p)$$

with λ_1 all pairwise different (if $p = 2$).

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$$P(V_1 \oplus V_2) = P(V_1) \otimes P(V_2) = \bigoplus_{\mu,\lambda} V^n_{c(\lambda_1)} \otimes (V^2_{\lambda_2} \otimes V^2_{\mu_2}) \otimes V^m_{c(\mu_1)}$$

Super duality

$$P(\mathbb{C}^n \otimes \mathbb{C}^{p|q}) = \begin{cases} \bigoplus_\lambda V_\lambda^n \otimes V_{\lambda'}^{p|q} & : \quad \mathrm{GL}_n \times \mathfrak{gl}(p|q) \\ \bigoplus_\mu V_\mu^n \otimes V_{\mu'}^{p|q} & : \quad \mathrm{Sp}_n \times \mathfrak{osp}(2p|2q) \end{cases}$$

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$$\mathrm{Hom}_{\mathfrak{g}'}(V_{\lambda'}, Res_{\mathfrak{g}'}^{\mathfrak{h}'}(V_{\mu'})) = \mathrm{Hom}_H(V_\mu, Res_H^G(V_\lambda))$$

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Via reciprocity, $Res^{\mathfrak{spo}(2p|2q)}_{\mathfrak{gl}(p|q)}(V_\lambda)$, $Res^{\mathfrak{osp}(2p|2q)}_{\mathfrak{gl}(p|q)}(V_\lambda)$